The Frequency of Cycles and Condorcet Inconsistency with IRV: in FairVote and Politbarometer Data

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Abstract

Instant Runoff Voting (IRV) or "the alternative vote" is now used by some countries and large cities in the US. Studies have concerned about a severe disadvantage of IRV, monotonicity failure, with theoretical approaches, but not much with data analysis. We address empirical analysis with two particular data sets, actual elections (FairVote) and surveys (Politbarometer). We estimate the frequency of Condorcet paradox and Condorcet inconsistent result of IRV, which is deeply related to the monotonicity criterion. We check the vote gap between the Condorcet winner and the Condorcet loser for Condorcet paradox, and that of between the Condorcet winner and a candidate who got fewer votes in two non-Condorcet winners by plurality rule for Condorcet inconsistency. About 1,000 synthetic elections, we found one Condorcet Paradox and twenty Condorcet inconsistencies (frequency of monotonicity failure is 2.15%). Our model provides 2.18~5.46% and 2.35~2.71% from two data sets.

1. Introduction

It is simple to understand, easy to process, and cost-saving (in money and time, compared to other rules). That explains why the plurality rule, even it is seemingly old-fashioned, is the second most used system in election for national legislatures.

Studies argued the weaknesses of the plurality rule and suggested other rules that to cure them. They are "supposed" to be better than the old one: more straightforward, cheaper, fairer, more democratic, more incentivize voters to be honest, and so forth. Researchers studied for a long time to evaluate different voting systems for criteria theoretically. Since other systems are rarely adopted in the real world, however, finding relevant data and empirical evidence to champion their theory was limited, and researchers lean on probabilistic models and simulations.

“Un”expectedly, from some moment alternative voting rules start to be taken account into. Specifically, some countries (Australia and Ireland) adopt Instant Runoff Voting (IRV) for national elections, and some states in the US also use the system for local elections. Additionally, in more than 50 US colleges and political party elections, IRV is also used. We use ranked-ballot data of US local elections, FairVote data, for this research.

Additionally, we use survey data, German Politbarometer. There is a section of the questionnaire; the participants are asked to rate integers on candidates. From the data, we set the rank of voters by score, and run "elections."

With two different data sets, we check the practical significance of Condorcet paradox, and Condorcet inconsistency. Then we suggest empirical oriented modeling, to estimate the frequency of those. Finding out the gap between the theory and the data is our main goal. The main idea of modeling are as follows.

Suppose a three-candidates election with a Condorcet winner and Condorcet loser, comparing two extreme cases the case that the winner beats by 51:49, and the case that the winner beats by 99:1. If few voters changed their ballots, there would be a turnover in the first case and the voting cycle occurs. On the other hand, a lot of voters need to change their ballots to make that turnover in the second case. In this sense, we regard the gap between votes that Condorcet winner and Condorcet loser get as an index of the voting cycle occurrence.

Eliminating the Condorcet winner under IRV is possible if the Condorcet winner gets the fewest votes in plurality rule. Therefore, the vote gap between that the Condorcet winner and a candidate who got fewer votes in two non-Condorcet winners indicates the likeliness of Condorcet inconsistency.

Contents

In the next section, we briefly introduce related materials and literature. Section 3 will describe the data set, provide basic statistics, and explain how we clean the data. After that, in each subsection. We check out relevant figures, illustrate our model, and report the results. Finally, in section 4, we summarize our work.

2. Preliminaries and Related Literatures

Condorcet Method and Condorcet criterion

The Condorcet method is a ranked-ballot voting rule that decides who wins the most in pairwise majority elections with all possible pairs. When a candidate beats all others one-on-one, the candidate is called Condorcet winner and considered the best candidate if one exists. When a voting system chooses the Condorcet winner as the winner of the election, we say the voting system satisfies Condorcet criterion, or it is Condorcet consistent, and it is one of the criteria to evaluate the voting system.

Instant Runoff Voting and Monotonicity failure

Instant Runoff Voting is one of the ranked-ballot voting rules and is also called Ranked Choice Voting. William Robert Ware proposes this rule. The process is as follows.

1. Add up the votes of the first ranked, find a candidate who wins the most.
2. If the candidate wins more than half of the total votes, then the candidate is the winner, and the process is done.
3. If not, eliminate a candidate with the fewest votes. The votes who set this eliminated candidate as their first-ranked, now the second-ranked is considered as their first-ranked, the third-ranked as the second, and continued.
4. Repeat

Under the plurality rule, especially when the two huge parties are ruling the politics, voters who prefer the third option are incentivized to choose one of two huge parties rather than revealing their honest first pick. Additionally, when the third party is politically close or similar to one of two huge parties, the third party might "steal" enough votes, affecting the win or loss of two parties (it is called the spoiler effect).

By its' design, IRV is supposed to make up for those shortcomings, but other new, different disadvantages pop up. Suppose two candidates A and B, and A is going to win the election if nothing happens. Then, suddenly some supporter of B changed their mind and cast their votes for A, and B wins and A loses the election in the end. How could one understand this far-fetched situation? Depending on the voting rule, this singular and counterintuitive outcome is feasible. This is called non-monotonicity or monotonicity failure. **Lepelley et al (1996)** defines the monotonicity paradoxes as follows

Paradox M1 (or the more-is-less paradox): the winner is ranked higher by one or more individuals (all else unchanged) and becomes a loser.

Paradox M2 (or the less-is-more paradox): a loser is ranked lower by one or more voters (all else unchanged) and becomes a winner.

Many researchers have given attention to this anomaly. The theoretical approach indicates that the event is mathematically plausible and commonly happens; **Felsenthal and Tideman (2013, 2014)** provided numerical examples of monotonicity failure under IRV. **Ornstein and Norman (2014)** and **Miller (2012, 2017)** calculated the frequency using probabilistic models.

Although the mathematics showed high possibilities, other studies argued that empirical evidence for non-monotonicity under IRV is quite rare**;** **Graham-Squire and Zayatz (2020)**. In addition, **Plassmann and Tideman (2014)** uses a theoretical approach and simulation, estimated much lower probability than previous studies.

Impartial Culture (IC) and Impartial Anonymous Culture (IAC) Models

There are two classic model, Impartial Culture (IC) and Impartial Anonymous Culture (IAC) model.

For *n* numbers of candidates, there exist *n*! numbers of fully specified (strictly) preference orderings. IC model assumes those *n!* preference orders are equally likely for each voter's preference.

Suppose *n* number of candidates, and *m* number of voters. Each voter's preference decided among *n!*, then a preference profile is a collection of *m* preference orders from each voter. IAC model assumes equal probability on different 'category (set of preference profiles that have same composition, ignore the arrangement)' of preference profiles.

Gehrlein (2002) provided the probability of voting cycle with IC and IAC conditions expect, under different conditions. For three-candidates and infinitely many voters case, each model induces 8.77% and 6.25%, respectively.

In the paper **(Lepelley et al, 1996)**, the authors calculated the limit probability of monotonicity failure, 5.74%, under IAC condition. They concluded 'it seems difficult to claim that monotonicity paradoxes are extremely rare and have no practical relevance.'

Spatial model

Starting from the idea of Hoteling-Downs model **(Hotelling, 1929; Downs, 1957)**, the spatial model or spatial election model was introduced. **Good and Tideman (1976)** describe the modeling method in detail. The model supposes the candidates or options are located in attribute space or policy space, and each voter has their ideal point for the location of a candidate or an option. The voters with various tastes rank all candidates based on their ideal points close to each alternative. We will take a simple, intuitive, and widely used one-dimensional setup and empirically analyze invalidity and how much departed when we have three candidates.

Ornstein and Norman (2014) calculated the frequency of nonmonotonicity based on the spatial model in two dimensions with different settings of the distribution of voters. They also provide necessary and sufficient conditions for monotonicity failure of three-candidate election under IRV. They conclude that “a lower bound estimated of 15% for competitive elections,” and the probability goes up to 51% depends on the setting.

Plassmann and Tideman (2014) simulated three-candidate elections based on a two-dimension spatial model. They counted the frequency of monotonicity failure under IRV, and it was 1%. A significant difference to ours is that we only count the “competitive” three candidates.

3. Analysis

Description of the Data

In this section, we will describe the data we use; FairVote and German Politbarometer.

The former is actual data of numerous local elections across the states[1]. The elections are sporadic; the earliest election is November 2004, the latest one is November 2020. The electorate system is Instant-runoff voting (IRV). The data is a rank-order-ballot of voters, not like information such a simple result of an election run by plurality rule, one could observe voters’ preference order on candidates fully or partially.

The latter is survey data which are freely available on the official website[2]. It is conducted

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[1] Including San Francisco, Cambridge, Minneapolis, Berkeley, Oakland, and New york city, more than 40 cities have been used or plan to use instant runoff voting during 2020~2022 (https://www.fairvote.org/data_on_rcv#research_snapshot).

[2] Freely available to the public at gesis website(https://www.gesis.org/en/elections-home/politbarometer) after sign up. These data come from surveys of German voters, conducted approximately monthly. The surveys contain many questions about demographic characteristics, region of residence, and weekly working hours, etc.

monthly (approximately); the data began in March 1977, ends in December 2017. Participants answered hundreds of questions for each survey, and we use a category of questions, a skalometer. For each question, the participant scores one of the integers from -5 to 5 to politicians. The pool consists of popular or famous politicians of the survey period.

We derive voters' preference order from the score data. For the research purpose, each survey is regarded as an election, and we assume that all surveys are independent of each other. We also assume that the participants would vote to comply with their evaluations. We are handling the survey data and the actual election data the same along.

In FairVote data, we have 172 observations. Three to 35 candidates are in each election (5.372 on average), and the elections range in size from 1,557 to 298,808 ballots (40253.198 on average). On the other hand, we use 811 surveys as observations in Politbarometer data. Four to 21 politicians are evaluated (10.924 on average), and the number of participants is from 192 to 4,494 (1133.057 on average) in each survey.

There is the Condorcet winner in all elections of FairVote, and there is not in 5 elections of Politbarometer (out of 811 elections, 0.617%). A cycle among the top three candidates (Condorcet ranking) happens only in Politbarometer, but just in one election (0.123%). We contrast the Condorcet winner and Instant-runoff voting winner (IRV), and count how often the IRV does NOT choose the Condorcet winner in each data set; they are 1 out of 172, and 19 out of 806, respectively.

Data Framework

Every election data of FairVote shows individual voters' first-choice, second-choice, and third-choice (or more than three, it varies). Each choice should be filled with one of the candidates, but there are some anomalous: “skipped,” “write-in” and “overvote.″

″Skipped” is as literally, when the voter did not make his/her choice for first-choice, second-choice, or third-choice. “Write-in” occurs when a voter wants to vote for someone not on the candidate list and writes the person’s name on a ballot. Finally, “Overvote” is the case that the voter gives the same rank to more than two candidates.

″Skipped,” “write-in” and “overvote” are removed and filled up by the following preferred candidate if it is possible. Specifically, A-[skipped]-B, A-[write-in]-B, or A-[overvote]-B will be considered as the voter prefer A the most and then B the second most, A-B or shortly AB. All weak preferences are not counted.

In the other data set, Politbarometer, we assume that the participants would vote precisely aligned on their evaluation. If the participant assigned the same score on multiple politicians, we considered the voter equally preferred or indifferent on them. If the participant did not assign any score on some politicians, we considered he would not rank for them in the election. For instance, if a participant assigns 4,-1, “blank”, 3, -1 for A, B, C, D, and E respectively, then it counts as A-D-[B and E] or shortly AD(BE).

Depending on the preference order we extract, we choose the top three candidates from the whole candidate pool by the Condorcet method in each election. In other words, three candidates win the most by the plurality rule, pairwise. We narrow the scope to the top three candidates because, firstly, the top cycle matters and three is the minimum for the cycle and compare the Condorcet method and IRV. Secondly, the lesser, the more convenient to handle.

For the cases that are impossible to define the top three candidates because of the cycles or ties, then break the cycle and tie by breaking the weakest link (comply on Ranked Pairs, **Tideman, 1987**) and picking one who is winning in their head-to-head election. We suppose the first candidates from above as the same as the Condorcet winner for later on.

In some elections, the top three candidates are not similarly competitive. For example, in an election for the city council member, district1, San Leandro, in 2010, the third candidate was ranked by only 0.757% of the total votes. In contrast, the first and second candidates were ranked by 84.088% and 72.278%. There were a couple of elections that had one dominating candidate and two "weak" candidates. In an election for the mayor of San Francisco in 2007, three top candidates were ranked by 89.312%, 20.113%, and 16.339%, respectively. They are not suitable for our research; we exclude eight elections from our observations of FairVote.

3.1. Estimating the Probability of the Top Cycle

We count the votes that the first candidate (the Condorcet winner) wins and the votes that the third candidate wins in their head-to-head election and define the "third-first ratio," which is former to latter. If the ratio gets closer to 1, only a few votes are needed by the third candidate to beat the first candidate. When the ratio is greater than 1, the third candidate beats the first candidate, and a cycle occurs.

While observing the FairVote data, we found out that the mean of the data increases gradually as the number of candidates increases, but the amount is diminishing, and the variance does not change significantly. Hence, we suppose that the third-first ratio follows the same distribution, but the support shifts depending on the number of the candidates.

We use the four-parameter beta distribution for convenience. The distribution could be fitted with numerous shapes and support, and one could get a good approximation of parameters by method of moments. The distribution has two shape parameters and the other two for the lower and upper bound. The probability density function is as follows.

$$\mathrm{f}(\mathrm{x}; \alpha, \beta, \mathrm{L}, \mathrm{U}) = \frac{1}{(U-L)^{\alpha+\beta-1}} \frac{(x-L)^{\alpha-1}(U-x)^{\beta-1}}{\mathbf{B}(\alpha, \beta)}$$

where

$$\mathbf{B}(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

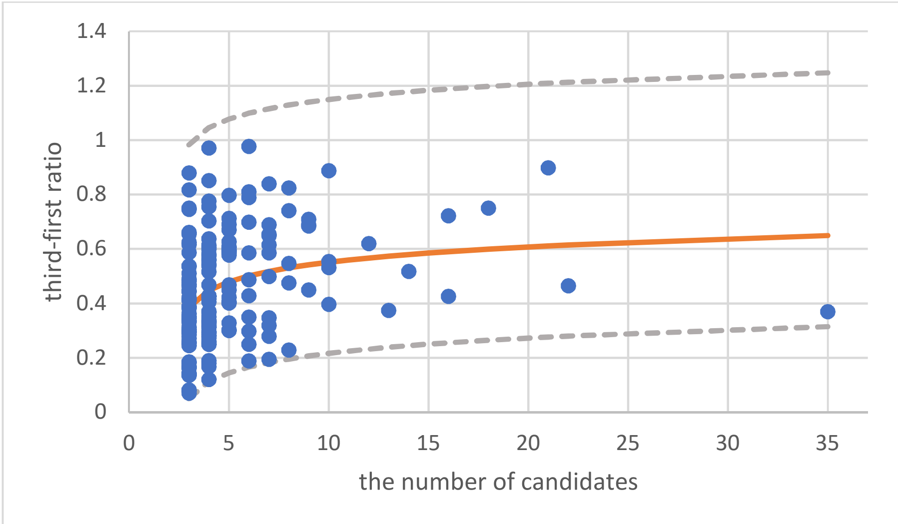
In our model, we need to define L (or U) as a function of the number of candidates. To capture the diminishing shifting of the support, we use the logarithm function.

$$\mathrm{L}(N; A, C, D) = A + C \cdot ln\left(\frac{N+D}{N_0+D}\right)$$

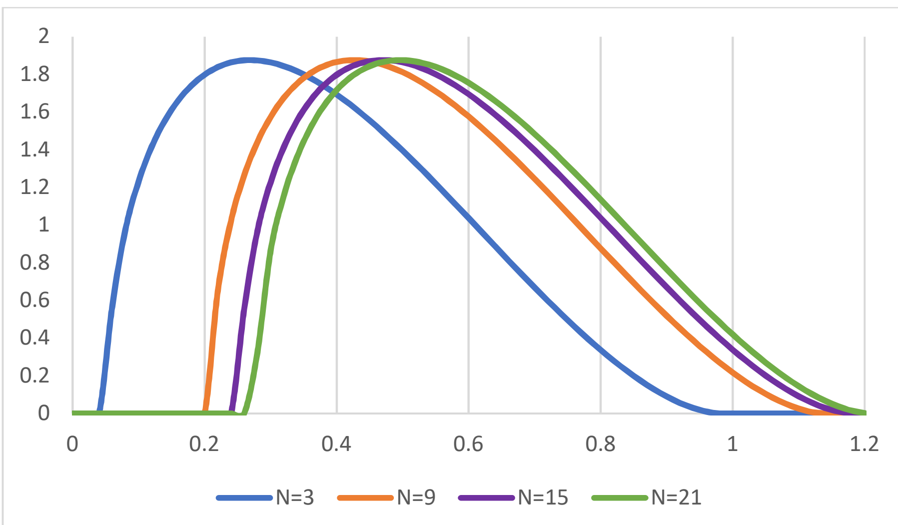
$$\mathrm{U}(N; A, C, D) = L + B$$

where N is the number of candidates, N_0 is the minimum of the candidates, A is the initial lower bound, B is the support length, and C and D are a coefficient and constant that capture support shifting.

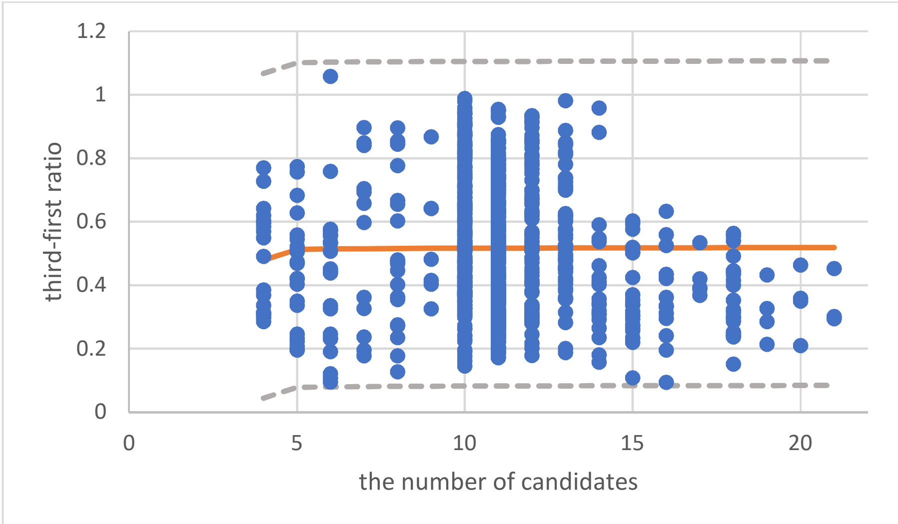
By maximum likelihood estimation, we take six parameters, $(\alpha, \beta, \mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})$.



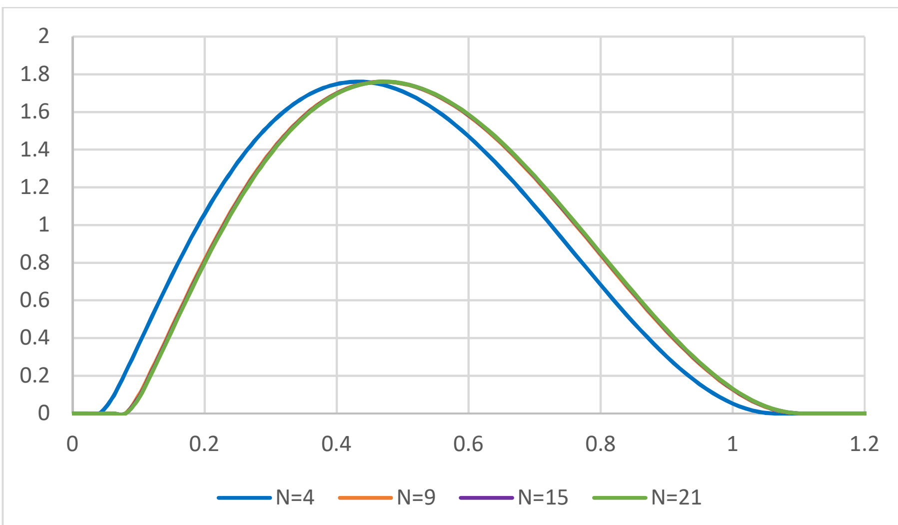
<figure 1A-(a). FairVote, scatter plot and supports of the estimated distributions>



<figure 1A-(b). FairVote, distributions depend on the number of candidates>



<figure 1B-(a). Politbarometer, scatter plot and supports of the estimated distributions>



<figure 1B-(b). Politbarometer, distributions depend on the number of candidates>

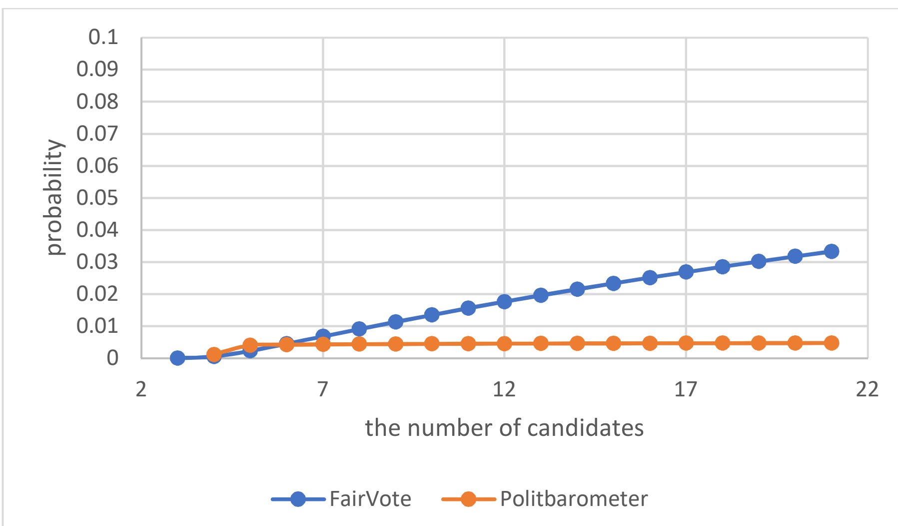
By integrating the pdf of the third-first ratio greater than 1, we have calculations of probability for each number of candidates. They are smaller than models under IC and IAC conditions expect, 0.0877 and 0.0625, respectively **(Gehrlein, 2002)**.

N	FairVote	Polit-barometer

N	FairVote	Polit-barometer

3	0	-
4	0.0005	0.0012
5	0.0023	0.0040
6	0.0045	0.0042
7	0.0068	0.0043
8	0.0091	0.0044
9	0.0113	0.0044
10	0.0135	0.0045
11	0.0156	0.0045
12	0.0176	0.0046
13	0.0196	0.0046
14	0.0215	0.0046
15	0.0233	0.0046
16	0.0251	0.0047
17	0.0269	0.0047
18	0.0285	0.0047
19	0.0302	0.0047
20	0.0317	0.0047
21	0.0333	0.0048

<table. 1. estimated probability of cycle>



<figure. 1C estimated probability of cycle>

3.2. Estimating the Frequency of Condorcet Inconsistent Outcomes with IRV.

IRV is known as a voting system that does not satisfy the Condorcet criterion. Our interest is predicting the frequency of the event based on empirical evidence.

We found the Condorcet winner differs from IRV winner in just one case out of 172 elections from FairVote (mayoral election of Burlington in 2009), and that of Politbarometer data is 19 cases out of 806 elections.

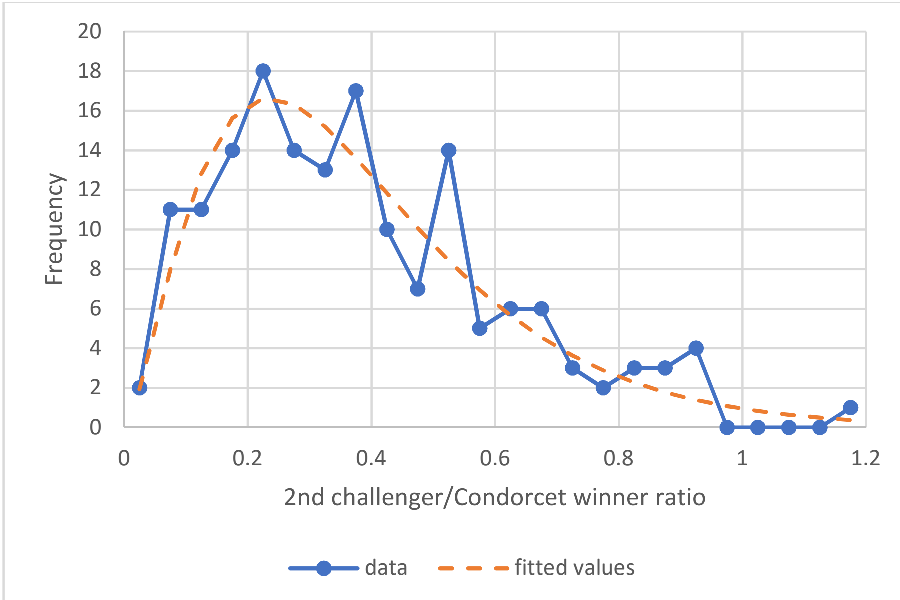
In these 20 "anomalies," 18 cases (1 from FairVote 17 from Politbarometer) are that the IRV winner comes the 2nd by Condorcet method, and the Condorcet winner comes the 3rd by IRV. The other two cases are similar but different from the 18 cases above.

In the survey of April 1987, the IRV winner comes the 2nd by Condorcet method as formers, but the Condorcet winner is a fourth candidate by IRV and not in the top three. In the survey of June 1996, there is the Condorcet winner who comes 3rd by IRV, and three candidates (including IRV winner) are tied for second place by Condorcet method, and the cycle occurs in them.

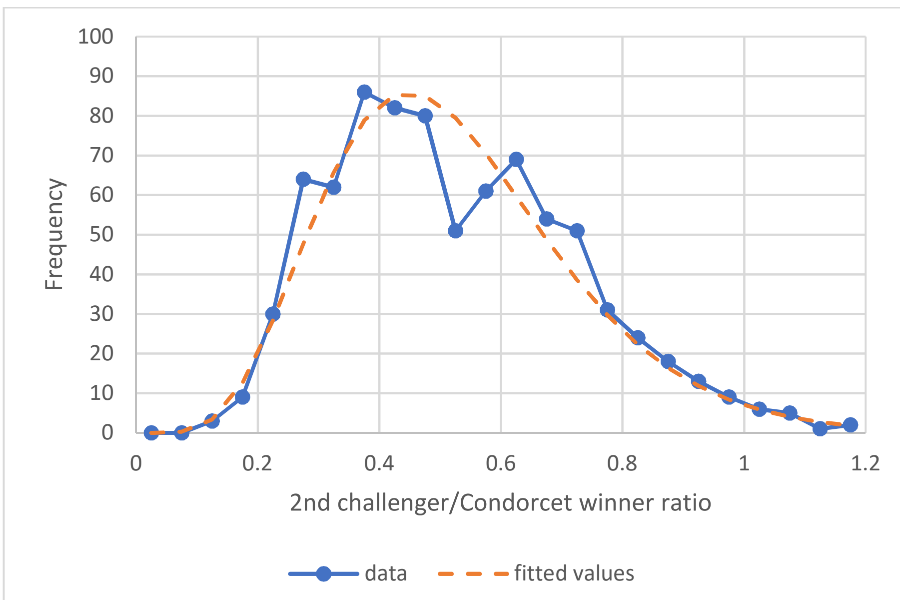
We count the votes that the Condorcet winner wins and the votes that the 2nd challenger wins in plurality voting with the top three candidates. Then we define the ratio of the second challenger/the Condorcet winner. The 2nd challenger is the candidate who wins fewer votes between the two candidates other than the Condorcet winner. When the ratio is close to 1, fewer votes are needed to get rid of the Condorcet winner by IRV. When it is greater than 1, the Condorcet winner will be eliminated by IRV with the three candidates or on the round with the three candidates remains.

Based on the shape of the histogram of the data, we decided to use generalized (three-parameter) gamma distribution and take three parameters by maximum likelihood estimation to fit the data.

$$\mathrm{f}(\mathrm{x}; \mathrm{a}, \mathrm{d}, \mathrm{p}) = \frac{p/a^d}{\Gamma(d/p)} x^{d-1} e^{-(x/a)^p}$$



<figure 2A. FairVote (a = 0.1477, d = 2.5955, p = 1)>



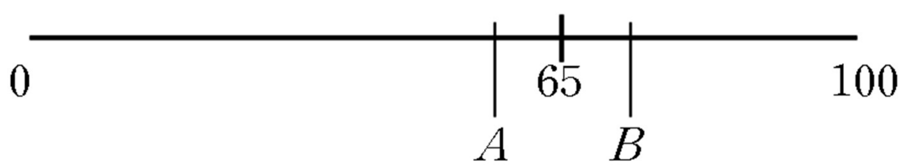
<figure 2B. Politbarometer (a = 0.0769, d = 6.8180, p = 1)>

With the estimated distribution, the ratio will be greater than 1 with probability 0.0213 and 0.0223, respectively. As we mentioned above, on the other hand, the data show 1 out of 172 (0.00581) and 19 out of 806 (0.02357).

3.3. One-dimensional Spatial Model for Three Candidates: Empirical Evidence

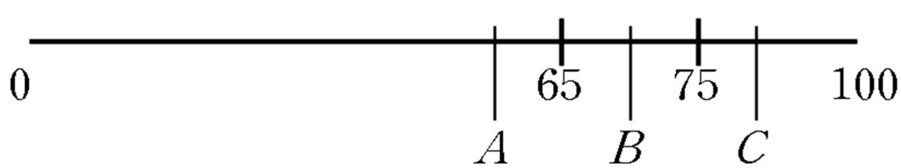
Spatial model, Three candidates in one dimensions

Suppose two candidates, and each candidate is located on a line depending on their political attributes. A line segment and the voters' ideal points are uniformly distributed. If there are two candidates A and B, and each of them obtains 65% and 35% of votes then,

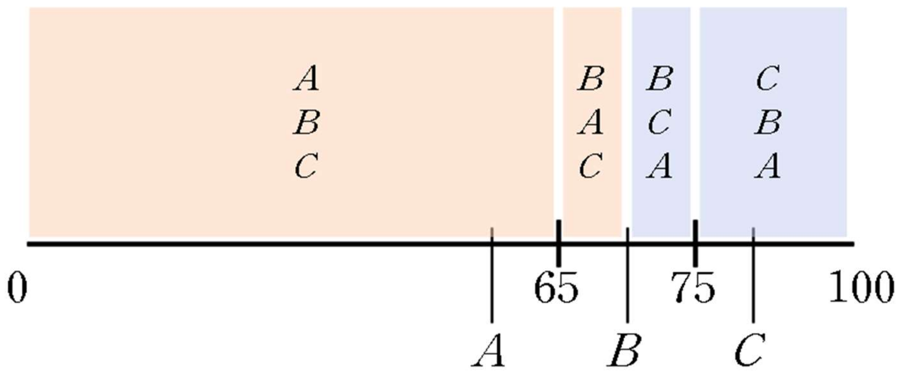


One could describe the election by putting A on 60 and B on 70 (or any pair that makes the middle point of two to be 65).

Now, suppose there are three candidates A, B, and C, and the result of head-to-head plurality rule; A beats B by 65:35, B beats C by 75:25, and A beats C by 70:30.



We fit the one-dimension spatial model by putting A on 60, B on 70, and C on 80. However, when we count the preference order of voters, the model reveals a flaw; The model is only capable of four preference orders. Thus, in the example, candidate B is located between A and C, and the voters who have preference order ACB and CAB cannot be represented.

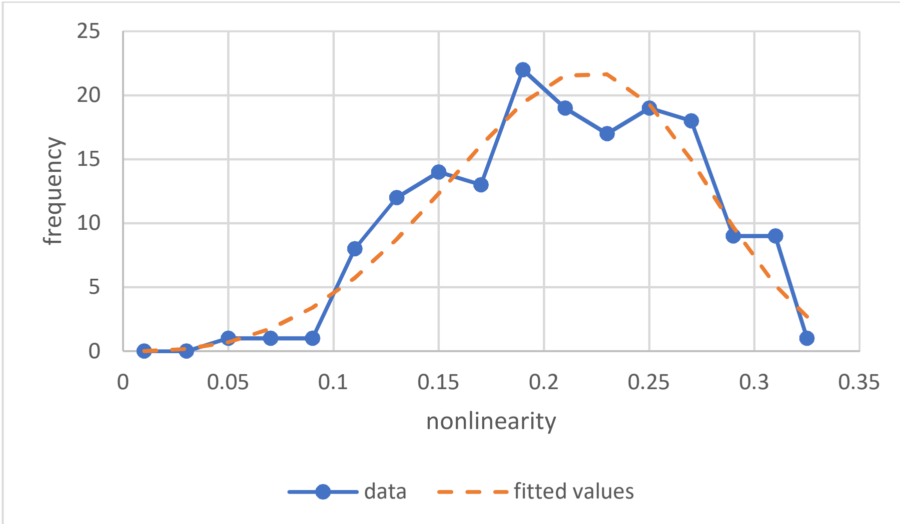


Estimation and Result

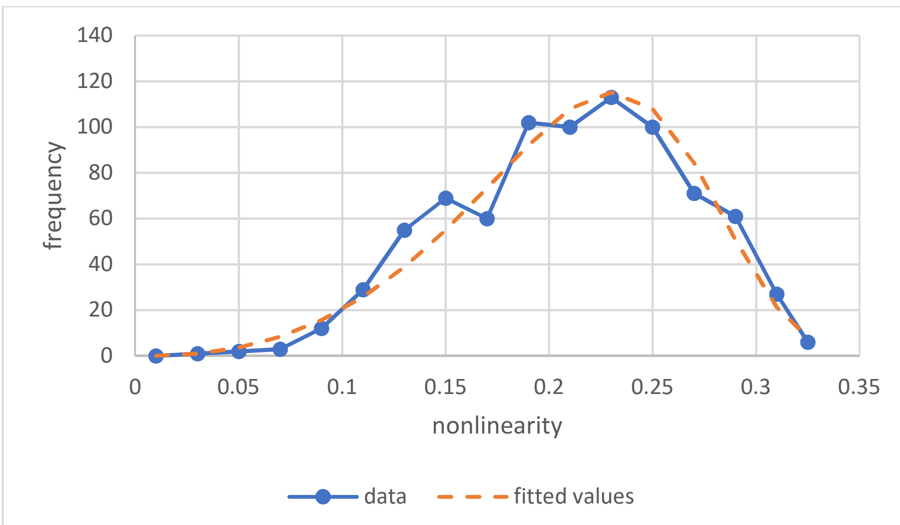
We cleaned the data, calculated the preference order of each voter, and counted the numbers for each type. By checking the share of each preference order, we see how well a one-dimensional spatial model reflects the data.

If an election with three candidates is perfectly fit by a one-dimensional setting, one (and only) candidate has no third-placed votes as candidate B in the previous example. Therefore, voters' least preferred candidates matter, and we hold only the votes with a fully specified preference on the top three candidates. Next, we count the third-placed votes for each candidate and define "nonlinearity," the smallest share of third-placed votes to measure how close the data is to the one-dimension spatial model. By definition, the nonlinearity is less than 1/3, and close 0 means a one-dimensional spatial model well captures it.

We use the generalized (3-parameter) gamma distribution to fit the data. Three parameters are found by the maximum likelihood method.

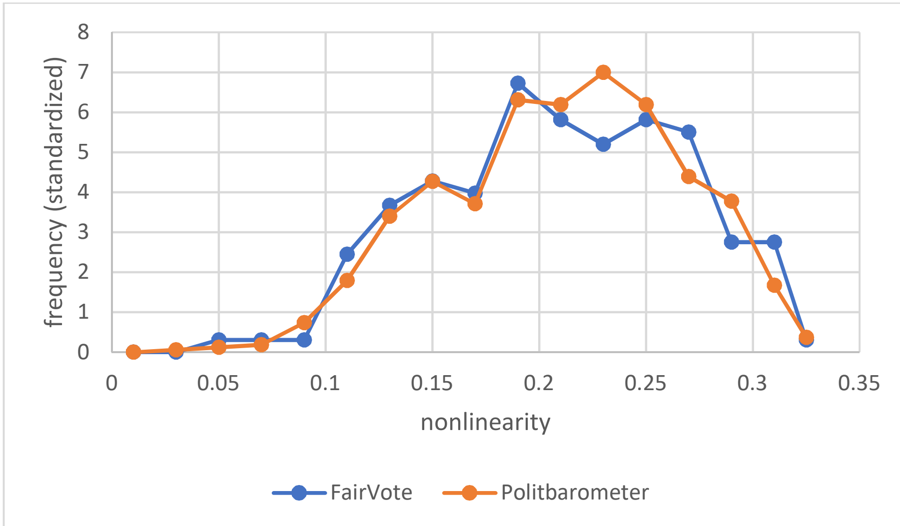


<figure 3A. FairVote (a = 0.2509, d = 3.6542, p = 5)>



<figure 3B. Politbarometer (a = 0.2680, d = 3.4906, p = 7)>

The minimum is 0.049 in FairVote and 0.038 in Politbarometer. About 95% of observations range from 0.1 to 0.32, and thus we claim the data do not support the one-dimensional spatial model. Therefore, our data show that using the one-dimension spatial model to evaluate voting rules can bring inaccurate estimation. Another finding is that a distribution of nonlinearity from two data sets seems not significantly different **<figure 3C>**.

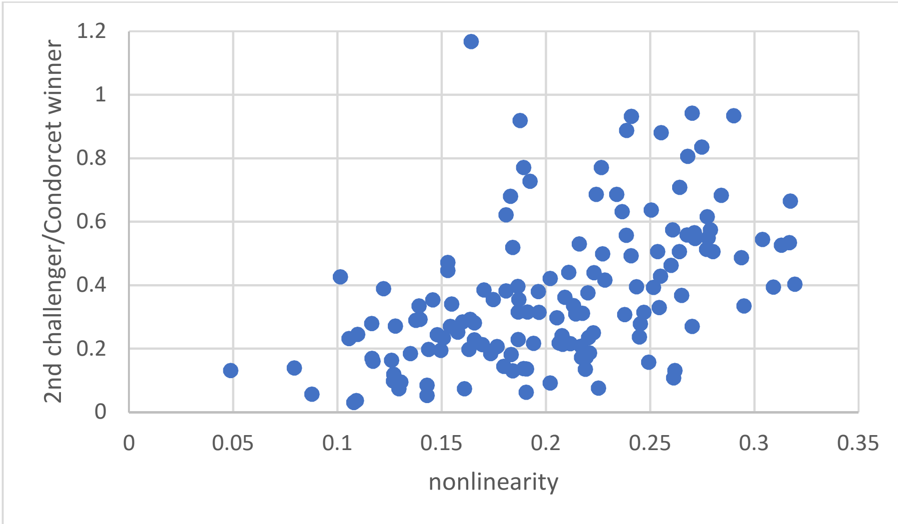


<figure 3C. distribution of nonlinearity FairVote and Politbarometer, standardized>

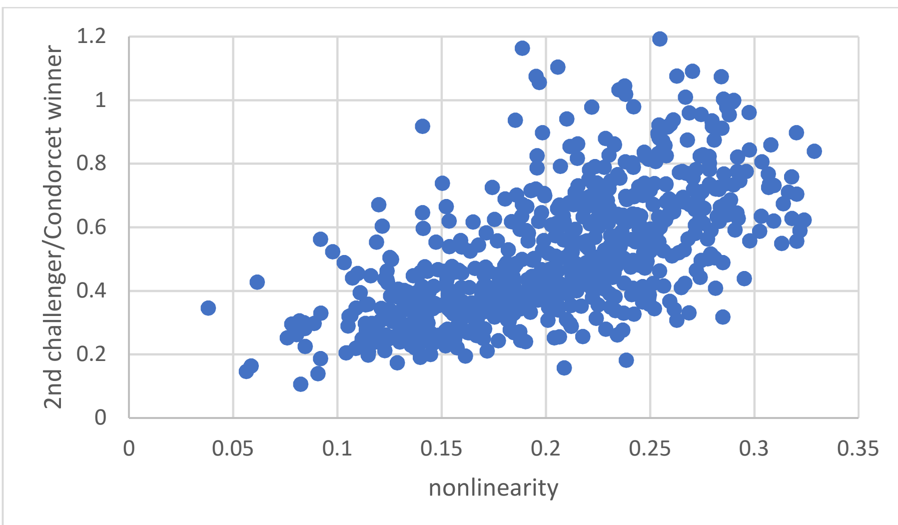
3.4 Nonlinearity and Failure of Condorcet criterion by IRV.

As the figure previously described, the candidate positioned between two other candidates or who gets the smallest third-placed vote is not necessarily the Condorcet winner and vice versa. However, according to the data we use, the Condorcet winner is the one who has the fewest third-placed votes in more than 85% of elections (FairVote 142 out of 164, Politbarometer 692 out of 811).

We use nonlinearity and the second challenger/Condorcet winner ratio from earlier sections and the following scatter plots show a non-linear positive relation between the two. It is not conclusive which leads from which, or it is also possible to accidentally move together.



<figure 4A. FairVote, linear correlation = 0.5078>



<figure 4B. Politbarometer, linear correlation = 0.6277>

First, let us assume that nonlinearity precedes the 2nd challenger-Condorcet winner ratio. When we adapt the one-dimension spatial model with three candidates, the Condorcet winner is the candidate closest to the median. The only possible scenario the candidate loses by IRV (or plurality rule) is squeezed by the other two candidates. In this sense, one could guess if the election can be described or approximated by the one-dimensional spatial model and the

Condorcet winner positioned in the middle, the Condorcet winner is more likely to lose by IRV. However, there is also another driving force which is the opposite. “The Condorcet winner who has the smallest third-placed votes” could mean the Condorcet winner is likely to be preferred the first most or the second most of the voters, and the probability of being beaten by the 2nd challenger will be lower.

Second, consider the other way around. For convenience sake, assume two-dimension attribute space, and voters’ distribution follows a bivariate normal distribution. Lower the 2nd challenger-Condorcet winner ratio means the 2nd challenger wins relatively few votes. Geometrically, this implies the 2nd challenger is located further from the mod of bivariate normal distribution. Consider two other candidates, the Condorcet winner and the 1st challenger, located near each other and close to the mod. Compare two situations: the 2nd challenger is located further from the mod, and the 2nd challenger is located closer to the mod. The positions of the three candidates will be more “linear” when the latter is the case. As the 2nd challenger getting closer to mod (or other two candidates), it becomes harder to fit by a line to three candidates.

4. Conclusion

There are several defects of the plurality rule other than just it is out-of-dated. However, it is still controversial which voting rule should be the next standard. Instant Runoff Voting seems to be a strong contender, and it is already used in many elections. In addition, researchers are interested in analyzing the IRS and potential (but severe) risk, monotonicity failure, which has been attractive topic for a long time. The anomaly is standing based on mathematics and theoretical modeling, and predicted possibility is high that it cannot be ignored. However, real-world data says the frequency is much lower than the expectation.

In this paper, with actual election data under IRV, we use surveys with scoring data, check the frequency of anomalies, and introduce empirical-based modeling. In our analysis, in 983 “elections” with competitive three candidates, there is only one voting cycle and 20 Condorcet inconsistencies. While excluding 5 cases that neither have voting cycle nor have Condorcet winner, the frequency of non-monotonicity is 2.15% (0.0215). On the other hand, our model provides 2.18~5.46% from actual election data, 2.35~2.71% from the survey data for estimating of voting cycle or Condorcet inconsistent outcome. In IRV, monotonicity failures can occur only when there exists voting cycle or Condorcet winner did not choose **(Ornstein and Norman, 2014)**, so estimation of monotonicity failure should be rarer than those figures from our model. Thus, the prediction is close to the actual data and far from what theories predict.

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